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- This allows proof by contradiction.
- A definite clause knowledge base is always consistent. This won't be true with the rules that imply *false*.

# Horn clauses

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- A Horn clause is either a definite clause or an integrity constraint.

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Example: If $\{c, d, e, f, g, h\}$ are the assumables

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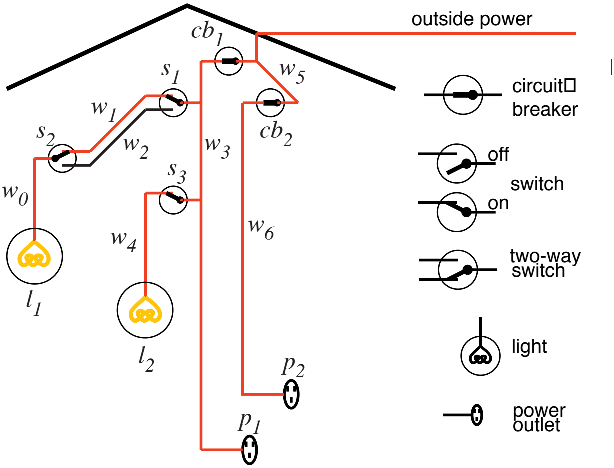
- Suppose switches $s_1$, $s_2$, and $s_3$ are all up:
  $up\_s_1.\ up\_s_2.\ up\_s_3.$

# Electrical Environment




in aipython.org, run code at the end of

`logicAssumables.py`

# Representing the Electrical Environment

$light\_l_1.$

$light\_l_2.$

$up\_s_1.$

$up\_s_2.$

$up\_s_3.$

$live\_outside.$

$lit\_l_1 \leftarrow live\_w_0 \wedge ok\_l_1.$

$live\_w_0 \leftarrow live\_w_1 \wedge up\_s_2 \wedge ok\_s_2.$

$live\_w_0 \leftarrow live\_w_2 \wedge down\_s_2 \wedge ok\_s_2.$

$live\_w_1 \leftarrow live\_w_3 \wedge up\_s_1 \wedge ok\_s_1.$

$live\_w_2 \leftarrow live\_w_3 \wedge down\_s_1 \wedge ok\_s_1.$

$lit\_l_2 \leftarrow live\_w_4 \wedge ok\_l_2.$

$live\_w_4 \leftarrow live\_w_3 \wedge up\_s_3 \wedge ok\_s_3.$

$live\_p_1 \leftarrow live\_w_3.$

$live\_w_3 \leftarrow live\_w_5 \wedge ok\_cb_1.$

$live\_p_2 \leftarrow live\_w_6.$

$live\_w_6 \leftarrow live\_w_5 \wedge ok\_cb_2.$

$live\_w_5 \leftarrow live\_outside.$

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- Either $cb_1$ is broken or there is one of six double faults.

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- Example: For the proceeding example there are seven minimal diagnoses: $\{ok\_cb_1\}$, $\{ok\_s_1, ok\_s_3\}$, $\{ok\_s_1, ok\_l_2\}$, $\{ok\_s_2, ok\_s_3\}, \ldots$

# Recall: top-down consequence finding

To solve the query $?q_1 \wedge \ldots \wedge q_k$:

> $ac :=$ "$yes \leftarrow q_1 \wedge \ldots \wedge q_k$"
> **repeat**
> > **select** atom $a_i$ from the body of $ac$;
> > **choose** clause $C$ from $KB$ with $a_i$ as head;
> > replace $a_i$ in the body of $ac$ by the body of $C$
>
> **until** $ac$ is an answer.

# Implementing conflict finding: top down

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- Stop when all of the atoms in the body of the generalised query are assumable:
    - this is a conflict

# Example

$$false \leftarrow a.$$
$$a \leftarrow b \,\&\, c.$$
$$b \leftarrow d.$$
$$b \leftarrow e.$$
$$c \leftarrow f.$$
$$c \leftarrow g.$$
$$e \leftarrow h \,\&\, w.$$
$$e \leftarrow g.$$
$$w \leftarrow f.$$

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# Bottom-up Conflict Finding

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- If $\langle a, A_1\rangle$ and $\langle a, A_2\rangle$ are in $C$, where $A_1 \subset A_2$, then $\langle a, A_2\rangle$ can be removed from $C$.
- If $\langle \mathit{false}, A_1\rangle$ and $\langle a, A_2\rangle$ are in $C$, where $A_1 \subseteq A_2$, then $\langle a, A_2\rangle$ can be removed from $C$.

# Bottom-up Conflict Finding Code

$C := \{\langle a, \{a\}\rangle : a \text{ is assumable }\};$
**repeat**
    **select** clause "$h \leftarrow b_1 \wedge \ldots \wedge b_m$" in $T$ such that
        $\langle b_i, A_i\rangle \in C$ for all $i$ and
        there is no $\langle h, A'\rangle \in C$ or $\langle \mathit{false}, A'\rangle \in C$
            such that $A' \subseteq A$ where $A = A_1 \cup \ldots \cup A_m$
    $C := C \cup \{\langle h, A\rangle\}$
    Remove any elements of $C$ that can now be pruned
**until** no more selections are possible

# Assumption-based Reasoning

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- In abduction an agent makes assumptions to explain observations. For example, it hypothesizes what could be wrong with a system to produce the observed symptoms.
- In default reasoning an agent makes assumptions of normality to make predictions. For example, the delivery robot may want to assume Mary is in her office, even if it isn't always true.

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Designing a meeting time with determining when it is.

# An Assumption-based Framework

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  Assume $F$ are Horn clauses.
- $H$ is a set of formulae called the possible hypotheses or assumables. Ground instance of the possible hypotheses can be assumed if consistent.

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  $D$ is an explanation of $g$ if $F \cup D \models g$ and $F \cup D \not\models \mathit{false}$.
  A minimal explanation is an explanation such that no strict subset is also an explanation.
- An extension of $\langle F, H\rangle$ is the set of logical consequences of $F$ and a maximal scenario of $\langle F, H\rangle$.

# Example

$a \leftarrow b \wedge c.$
$b \leftarrow e.$
$b \leftarrow h.$
$c \leftarrow g.$
$c \leftarrow f.$
$d \leftarrow g.$
$false \leftarrow e \wedge d.$
$f \leftarrow h \wedge m.$
assumable $e, h, g, m, n.$

- Is $\{e, m, n\}$ a scenario?

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$c \leftarrow f.$
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$false \leftarrow e \wedge d.$
$f \leftarrow h \wedge m.$
assumable $e, h, g, m, n$.

- Is $\{e, m, n\}$ a scenario? yes
- Is $\{e, g, m\}$ a scenario. no
- Is $\{h, m\}$ an explanation for $a$. yes
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# Example

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Give observations, we typically do abduction, then default reasoning to find consequences.

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To find assumables to imply the query $?q_1 \wedge \ldots \wedge q_k$:

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To find an explanation of query $?q_1 \wedge \ldots \wedge q_k$:

- find assumables to imply $?q_1 \wedge \ldots \wedge q_k$
- ensure that no subset of the assumables found implies *false*