Missing Data

- You cannot just ignore missing data unless you know it is missing at random.
- Is the reason data is missing correlated with something of interest?
- Example: data in a clinical trial to test a drug may be missing because:
  - the patient dies
  - the patient had severe side effects
  - the patient was cured
  - the patient had to visit a sick relative.

  — ignoring some of these may make the drug look better or worse than it is.
- In general you need to model why data is missing.

Missing Data

- Suppose there is a drug claimed to treat a disease.
- The drug does not actually affect the disease or its symptom, but makes sick people sicker.
- Suppose patients were randomly assigned the drug or a placebo, but the sickest people dropped out of the study, because they become too sick to participate.
- What happens if the missing data (from patients who dropped out) is ignored?
- It looks like the treatment works; there are fewer sick people among the people who took the treatment and remained in the study!

Handling missing data requires more than a probabilistic model that models correlation. It requires a causal model of how the data is missing.

Missingness graph

A missingness graph, or *m*-graph, is a causal model of data where some values might be missing.

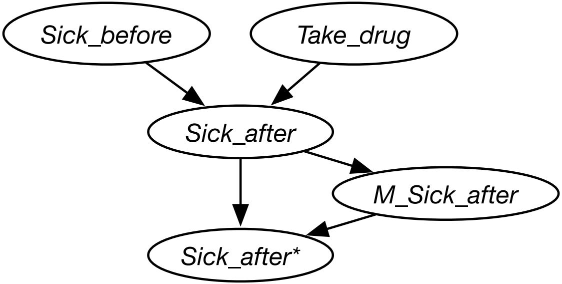
- Given a causal network, the *m*-graph contains all variables of the original graph with the same parents.
- For each variable V that could be observed with some values missing, the *m*-graph contains two extra variables:
  - Boolean variable M_V is true when V's value is missing. The parents of M_V are the variables missingness depends on.
  - Variable V^*, with domain $dom(V) \cup \{missing\}$ *missing* is a new value (not in the domain of V) V and M_V and the parents of V^*, with:

$$P(V^*{=}missing \mid M\_V{=}true) = 1$$
$$P(V^*{=}v \mid M\_V{=}false \land V{=}v) = 1.$$

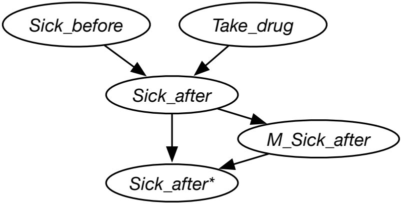
- If V is observed to be v, $V^*{=}v$ is conditioned on. If the value for V is missing, $V^*{=}missing$ is conditioned on.
- V^* is always observed.

Example *m*-graph

A drug that just makes people sicker and so drop out, giving missing data.
Missingness depends on whether they are sick after:



Training with Expectation Maximization



This could be trained using expectation maximization (EM) with *Sick_after* unobserved, *however*:

- There are many distributions consistent with the data:
  all of the unobserved could be very sick after
  none could be sick after taking the drug.
- EM could converge to any of these.
- EM makes up fiction about those with missing data.
- We need to determine why the data is missing.

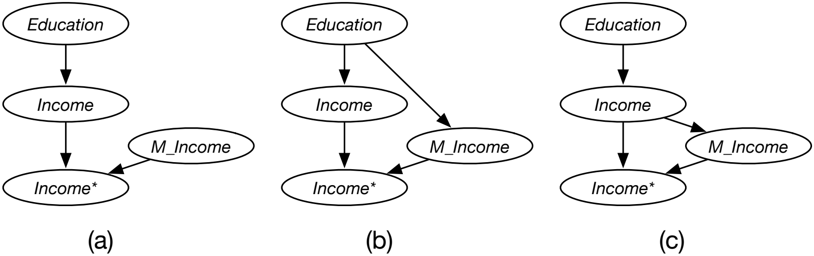
Recoverability

- A distribution is recoverable or identifiable from missing data if the distribution can be accurately measured from the data, even with parts of the data missing.
- Data for V is missing completely at random (MCAR) if V and M_V are independent. Missing data can be ignored.
- Variable Y is missing at random (MAR), when Y is independent of M_Y given observed variables V_o.

$$P(Y \mid V_o, M\_Y) = P(Y \mid V_o)$$

  then $P(Y, V_o) = P(Y \mid V_o, M\_Y{=}\mathit{false})P(V_o)$
- In other cases (e.g., previous case) the distribution may not be recoverable, depending on the graph structure.

Recoverability



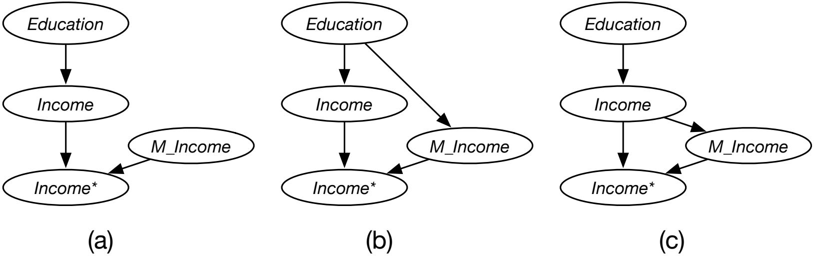
Education is observed but *Income* might have missing values:

(a) completely at random

(b) missing at random

(c) missing not at random

Recoverability

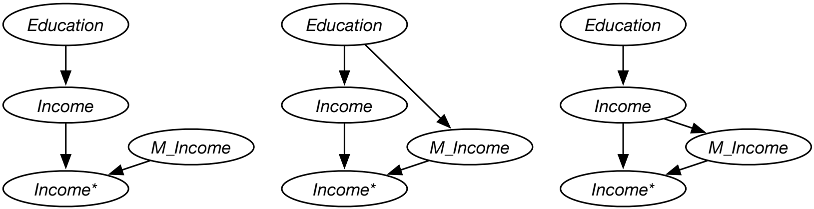


Education is observed but *Income* might have missing values:

(a) completely at random

$$P(Income, Education)$$
$$= P(Income^*, Education \mid M\_Income = false)$$

Recoverability



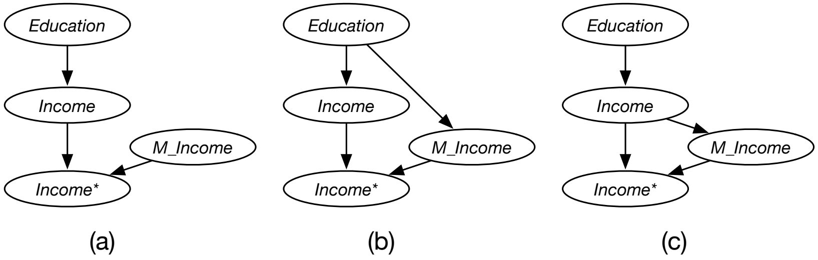
(a) (b) (c)

Education is observed but *Income* might have missing values:

(b) missing at random

$$
\begin{aligned}
&P(Income, Education) \\
&\quad = P(Income \mid Education) * P(Education) \\
&\quad = P(Income \mid Education \land M\_Income = false) * P(Education) \\
&\quad = P(Income^* \mid Education \land M\_Income = false) * P(Education)
\end{aligned}
$$

Recoverability



Education is observed but *Income* might have missing values:

(c) missing not at random (MNAR).

- In this graph, the relationship between income and education cannot be estimated from data.
- EM (and related algorithms) converge to fiction.
- In some cases of MNAR, probabilities can be computed, depending on the graph structure.