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- Example: assume that a database of what students are enrolled in a course is complete. We don't want to have to state all negative enrolment facts!
- The definite clause language is monotonic: adding clauses can't invalidate a previous conclusion.
- Under the complete knowledge assumption, the system is non-monotonic: adding clauses can invalidate a previous conclusion.

Equality

Equality is a special predicate symbol with a standard domain-independent intended interpretation.

- Suppose interpretation $I = \langle D, \phi, \pi \rangle$.
- t_1 and t_2 are ground terms then $t_1 = t_2$ is true in interpretation I if t_1 and t_2 denote the same individual. That is, $t_1 = t_2$ if $\phi(t_1)$ is the same as $\phi(t_2)$.

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- Example:
  $D = \{\text{✄}, \text{☎}, \text{✎}\}$.
  $\phi(phone) = \text{☎}$, $\phi(pencil) = \text{✎}$, $\phi(telephone) = \text{☎}$
  What equalities and inequalities hold?

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  $D = \{$✂, ☎, ✎$\}$.
  $\phi(\mathit{phone}) =$ ☎, $\phi(\mathit{pencil}) =$ ✎, $\phi(\mathit{telephone}) =$ ☎
  What equalities and inequalities hold?
  $\mathit{phone} = \mathit{telephone}$, $\mathit{phone} = \mathit{phone}$, $\mathit{pencil} = \mathit{pencil}$, $\mathit{telephone} = \mathit{telephone}$
  $\mathit{pencil} \neq \mathit{phone}$, $\mathit{pencil} \neq \mathit{telephone}$

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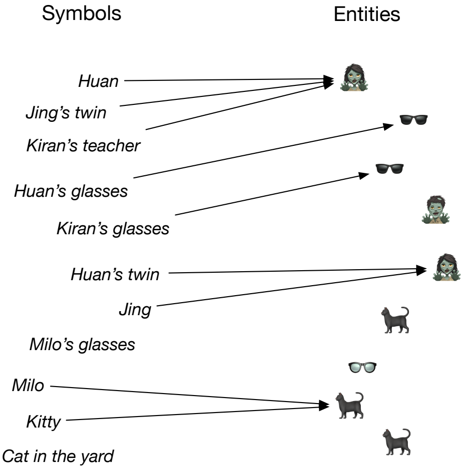
  What equalities and inequalities hold?
  $phone = telephone$, $phone = phone$, $pencil = pencil$,
  $telephone = telephone$
  $pencil \neq phone$, $pencil \neq telephone$
- Equality does not mean similarity!

Equality



Symbols

Entities

Huan

Jing's twin

Kiran's teacher

Huan's glasses

Kiran's glasses

Huan's twin

Jing

Milo's glasses

Milo

Kitty

Cat in the yard

Jing is Huan's twin. (=)

Jing is not Kiran's teacher. ($\neq$)

Properties of Equality

Equality is:

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For each n-ary function symbol f

$$f(X_1, \ldots, X_n) = f(Y_1, \ldots, Y_n) \text{ if } X_1 = Y_1 \text{ and } \cdots \text{ and } X_n = Y_n.$$

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Unique Names Assumption

- Suppose the only clauses for *enrolled* are

$$enrolled(sam, cs222)$$

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  - Inequalities:

  $$sam \neq chris \wedge cs873 \neq cs222$$

- The unique names assumption (UNA) is the assumption that distinct ground terms denote different individuals.

Inequality as a subgoal

- What should the following query return?

$$?- \quad X \neq 4.$$

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- Prolog has 3 different inequalities that differ on examples like these:

  `\==` `\=` `dif()`

  They differ in cases where there are free variables, and terms unify but are not identical.

Prolog's 3 implementations of not-equals

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  ```
  \==        \=       dif()
  ```

  which give same answers for variable-free queries, or when both sides are identical

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- They give different answers when there is a free variable.
  `\==` means "not identical". `a \== X` succeeds
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  `dif` is less procedural and more logical

Implementing dif

- $dif(X, Y)$
    - all instances fail when

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Example of dif

```
passed_two_courses(S) :-
    dif(C1,C2),
    passed(S, C1),
    passed(S, C2).
passed(S,C) :-
    grade(S,C,M),
    M >= 50.
grade(sam,engl101,87).
grade(sam,phys191,89).
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use_module(library(clpfd)).
    % https://www.swi-prolog.org/man/clpfd.html
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Constraint programming systems provide sophisticated constraint solving.

Completion of a knowledge base: propositional case

- Suppose the rules for atom a are

$$\begin{array}{l} a \leftarrow b_1. \\ \quad\vdots \\ a \leftarrow b_n. \end{array}$$

  equivalently $a \leftarrow b_1 \vee \ldots \vee b_n$.

- Under the Complete Knowledge Assumption, if a is true, one of the b_i must be true:

$$a \rightarrow b_1 \vee \ldots \vee b_n.$$

- Thus, the clauses for a mean

$$a \leftrightarrow b_1 \vee \ldots \vee b_n$$

Clark Normal Form

The Clark normal form of the clause

$$p(t_1, \ldots, t_k) \leftarrow B.$$

is the clause

$$p(V_1, \ldots, V_k) \leftarrow \exists W_1 \ldots \exists W_m \; V_1 = t_1 \wedge \ldots \wedge V_k = t_k \wedge B.$$

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- $V_1, \ldots, V_k$ are k variables that did not appear in the original clause
- $W_1, \ldots, W_m$ are the original variables in the clause.
- When the clause is an atomic clause, B is *true*.
- Often can be simplified by replacing $\exists W \; V = W \land p(W)$ with $P(V)$.

Clark normal form

For the clauses

$$student(mary).$$

$$student(sam).$$

$$student(X) \leftarrow undergrad(X).$$

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$$student(V) \leftarrow \exists X \ V = X \land undergrad(X).$$

Clark's Completion

Suppose all of the clauses for p are put into Clark normal form, with the same set of introduced variables, giving

$$p(V_1, \ldots, V_k) \leftarrow B_1.$$

$$\vdots$$

$$p(V_1, \ldots, V_k) \leftarrow B_n.$$

which is equivalent to

$$p(V_1, \ldots, V_k) \leftarrow B_1 \vee \ldots \vee B_n.$$

Clark's completion of predicate p is the equivalence

$$\forall V_1 \ldots \forall V_k \ p(V_1, \ldots, V_k) \leftrightarrow B_1 \vee \ldots \vee B_n$$

If there are no clauses for p,

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Clark's completion of predicate p is the equivalence

$$\forall V_1 \ldots \forall V_k \; p(V_1, \ldots, V_k) \leftrightarrow B_1 \vee \ldots \vee B_n$$

If there are no clauses for p, the completion results in

$$\forall V_1 \ldots \forall V_k \; p(V_1, \ldots, V_k) \leftrightarrow \mathit{false}$$

Clark's completion of a knowledge base consists of the completion of every predicate symbol along the unique names assumption.

Completion Example

Consider the recursive definition:

$$
\begin{aligned}
& passed\_each([\,], St, MinPass).\\
& passed\_each([C|R], St, MinPass) \leftarrow\\
& \qquad passed(St, C, MinPass) \land\\
& \qquad passed\_each(R, St, MinPass).
\end{aligned}
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In Clark normal form, this can be written as

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In Clark normal form, this can be written as

$$passed\_each(L, S, M) \leftarrow L = [\,].$$

$$\begin{aligned} & passed\_each(L, S, M) \leftarrow \\ & \qquad \exists C\ \exists R\ \ L = [C|R] \land passed(S, C, M) \land passed\_each(R, S, M). \end{aligned}$$

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$$\begin{aligned} &\forall L\ \forall S\ \forall M\ passed\_each(L, S, M) \leftrightarrow L = [\,] \lor \\ &\quad \exists C\ \exists R\ L = [C|R] \land passed(S, C, M) \land passed\_each(R, S, M). \end{aligned}$$

Clark's Completion of a KB

- Clark's completion of a knowledge base consists of the completion of every predicate.
- The completion of an *n*-ary predicate *p* with no clauses is $p(V_1, \ldots, V_n) \leftrightarrow \mathit{false}$.
- You can interpret negations in the body of clauses. $\sim a$ means *a* is false under the complete knowledge assumption. $\sim a$ is replaced by $\neg a$ in the completion. This is negation as failure.

Completion example

$p \leftarrow q \wedge \sim r.$

$p \leftarrow s.$

$q \leftarrow \sim s.$

$r \leftarrow \sim t.$

$t.$

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Completion:

$$p \leftrightarrow q \wedge \neg r \vee s.$$

$$q \leftrightarrow \neg s.$$

$$r \leftrightarrow \neg t.$$

$$t \leftrightarrow true.$$

$$s \leftrightarrow w.$$

$$w \leftrightarrow false.$$

Defining *empty_course*

Given database of:

- $course(C)$ that is true if C is a course
- $enrolled(S, C)$ that is true if student S is enrolled in course C.

Define $empty\_course(C)$ that is true if there are no students enrolled in course C.

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- Using negation as failure, $empty\_course(C)$ can be defined by

$$empty\_course(C) \leftarrow course(C) \land \sim has\_enrollment(C).$$

$$has\_enrollment(C) \leftarrow enrolled(S, C).$$

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- The completion of this is:

$$\forall C \ empty\_course(C) \iff course(C) \wedge \neg has\_enrollment(C).$$

$$\forall C \ has\_enrollment(C) \iff \exists S \ enrolled(S, C).$$

Bottom-up negation as failure interpreter

```
$C := \{\}$;
repeat
    either
        select $r \in KB$ such that
            $r$ is "$h \leftarrow b_1 \wedge \ldots \wedge b_m$"
            $b_i \in C$ for all $i$, and
            $h \notin C$;
        $C := C \cup \{h\}$
    or
```

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repeat
    either
        select $r \in KB$ such that
            $r$ is "$h \leftarrow b_1 \wedge \ldots \wedge b_m$"
            $b_i \in C$ for all $i$, and
            $h \notin C$;
        $C := C \cup \{h\}$
    or
        select $h$ such that for every rule "$h \leftarrow b_1 \wedge \ldots \wedge b_m$" $\in KB$
                either for some $b_i, \sim b_i \in C$
                or some $b_i = \sim g$ and $g \in C$
        $C := C \cup \{\sim h\}$
until no more selections are possible
```

Negation as failure example

$$p \leftarrow q \wedge \sim r.$$

$$p \leftarrow s.$$

$$q \leftarrow \sim s.$$

$$r \leftarrow \sim t.$$

$$t.$$

$$s \leftarrow w.$$

Top-Down negation as failure proof procedure

- If the proof for a fails, you can conclude $\sim a$.
- Failure can be defined recursively:
  Suppose you have rules for atom a:

  $$a \leftarrow b_1$$
  $$\vdots$$
  $$a \leftarrow b_n$$

  If each body b_i fails, a fails.
- A body fails if one of the conjuncts in the body fails.
- Note that you need *finite* failure. Example $p \leftarrow p$.

Floundering

$p(X) \leftarrow \sim q(X) \wedge r(X).$

$q(a).$

$q(b).$

$r(d).$

ask $p(X).$

- What is the answer to the query?

Floundering

$p(X) \leftarrow \sim q(X) \wedge r(X).$

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ask $p(X).$

- What is the answer to the query?
- How can a top-down proof procedure find the answer?

Floundering

$p(X) \leftarrow \sim q(X) \land r(X).$
$q(a).$
$q(b).$
$r(d).$
ask $p(X).$

- What is the answer to the query?
- How can a top-down proof procedure find the answer?
- Delay the subgoal until it is bound enough.
  Sometimes it never gets bound enough — "floundering".

Problematic Cases

$p(X) \leftarrow \sim q(X)$

$q(X) \leftarrow \sim r(X)$

$r(a)$

ask $p(X)$.

- What is the answer?

Problematic Cases

$p(X) \leftarrow \sim q(X)$
$q(X) \leftarrow \sim r(X)$
$r(a)$
ask $p(X)$.

- What is the answer?
- What does delaying do?

Problematic Cases

$p(X) \leftarrow \sim q(X)$

$q(X) \leftarrow \sim r(X)$

$r(a)$

ask $p(X)$.

- What is the answer?
- What does delaying do?
- How can this be implemented?